Math 2FM3, Tutorial 2

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Nominal Interest Rate

- $1+i=[1+i^{(m)}/m]^m$;
- $i^{(m)} = m[(1+i)^{1/m} - 1]$;
- the limit of $i^{(m)}$ is $\ln(1+i)$.

Effective Annual Rate of Discount

- Define $d=(A(1)-A(0))/A(1)$
- $A(0)=A(1)(1-d)$
- $A(n)=A(0)(1-d)^{-n}$

- Relations among d, v and i.
- $1-d=v=(1+i)^{-1}$

The Force of Interest

- $\delta(t)=A'(t)/A(t)$
- For simple interest rate:
- $A'(t)=A(0)*i$, then $\delta(t)=(A(0)*i)/(A(0)*(1+it))=i/(1+it)$
- For compound interest rate:
- $A'(t)=A(0)(1+i)^t \ln(1+i)$, then $\delta(t)=\ln(1+i)$.

Ex 1.4.5

- Smith receives income from his investments in Japanese currency (yen). Smith does not convert the yen to dollars, but invests the yen in a term deposit that pays interest in yen. He finds a bank that will issue such a term deposit, but it charges a 1% commission on each initial placement and on each rollover. The current interest rate on yen deposits is a nominal annual rate of 3.25% convertible quarterly for a 3-month deposit. To keep his yen available, Smith decides to roll over the deposit every 3 months. What is the effective annual after-commission rate that Smith earns?

- $i^{(4)} = 3.25\%$,
- each commission is 0.01.
- $A(12)=A(0)[0.99(1+i^{(4)}/4)]^4 =0.992198*A(0)$
- Then the effective rate is $(A(12)-A(0))/A(0)=0.992198-1=-0.0078$

Ex 1.5.5

- Bruce and Robbie each open up new bank accounts at time 0. Bruce deposits 100 into his bank account, and Robbie deposits 50 into his. Each account earns an effective annual discount rate of d. The amount of interest earned in Bruce’s account during the 11^{th} year is equal to X. The amount of interest earned in Robbie’s account during the 17^{th} year is also equal to X. Calculate X.

- $A_B\ (0)=100,\ A_R\ (0)=50$
- $X=A_B\ (11)-A_B\ (10)=A_R\ (17)-A_R\ (16)$
- Since $A(n)=A(0)(1-d)^{-n}$
- $100[(1-d)^{-11}-(1-d)^{-10}]=50[(1-d)^{-17}-(1-d)^{-16}]$
- then we get $d=0.1091$
- $X=38.9$

Ex 1.6.6

- Bruce deposits 100 into a bank account. His account is credited at a nominal rate i convertible semiannually. At the same time, Peter deposits 100 into a separate account. Peter's account is credited interest at a force of interest of δ. After 7.25 years, the value of each account is 200. Calculate i-δ.

- For both of them, $A(t)=200$, $A(0)=100$, $t=7.25$
- For Bruce:
- $A(t)=A(0)([1+i^{(m)}/m]^m)^t$,
- $200=100([1+i/2]^2)^{7.25}$
- $i=0.0979$

- For Peter:
- $A(t)=A(0)(1+i)^t =A(0)e^{\delta t}$
- $200=100e^{\delta*7.25}$
- $\delta=0.0956$
- $i-\delta=0.23\%$.